

Modified Newtonian Dynamics with Inverse Dissipation

Potential as an Alternative to Dark Matter and Dark Energy

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Abstract

This paper introduces the inverse dissipation potential into the Newtonian Dynamic equation and studies the motion equations of the objects in the isolated gravitational system. It is found that at large scales it can derive the dynamical equation of cosmic expansion similar to the Λ CDM model and yield the flat rotation curves for spiral galaxy. Different from the usual dark matter models, the derived flat rotation curves are the result of time accumulation rather than the direct action of mechanics. And the Tully-Fisher relationship is also discussed, it is found that the basic constant a_0 in the MOND model and the form of the function μ have a clear corresponding physical significance in the model of this paper.

Keywords

dark matter, dark energy, Newtonian dynamic equation, inverse dissipation, MOND

1. Introduction

Dark matter and dark energy are two major problems in today's cosmology [1-4]. These two "invisible" substances show two total distinct effects that dark matter produces gravity, while dark energy produces anti-gravity. Various theoretical models, based on general relativity, are proposed to explain these two phenomena. Obviously, there are two different modification methods. By changing the left side of the Einstein field equation, the time-space geometry itself is modified and form the theories represented by $f(R)$ gravity theory [5,6], brane-world gravity theory [7], MOND [8,9], etc. The other is to modify the right side of the Einstein field equation with additional fields or matter within the time-space, such as Λ CDM [10], Quintessence [11], phantom [12,13], etc.

However, none of the present theories could solve all problems perfectly, especially, since the search for dark matter particles and the measurement of dark energy have found nothing [14,15], it prompted us to think that perhaps modifying the gravity equation was an option to try. And it can be seen in Ref. [16-21] that some scholars have done a lot of work in this area.

Among the various models that replace dark matter, the MOND model proposed by M. Milgrom is a popularly discussed one. This model presented an empirical formula, in which a basic constant a_0 is introduced to explain some observational phenomena [22-28], for example, it can explain the Tully-Fisher relationship well [24,25]. However, the model still has some problems in explaining other observation data, such as the mass deviation in the cluster of galaxies [29], and the theory itself as it violates the basic law of momentum conservation [30].

In short, the MOND theory is not perfect at the time of its submission. Therefore,

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based on the original MOND theory, Bekenstein et al., TG Zlosnik et al. and M. Milgrom proposed the relativistic MOND theory, namely Tensor-Vector-Scalar Theory (TeVeS) [20,26], Einstein-Aether theory [31] and subsequently Bimetric MOND Theory [32,33], respectively. Although these theories greatly enrich the content of MOND, problems still exist [34,35].

The partial success of MOND theory and its extension theories vaguely implies that it maybe stems from a more basic theoretical system. Hence, this paper, based on the theory of non-relativistic Newtonian dynamics, utilizes the theoretical method of correcting inertia [37] by introducing the dissipative potential to Newton's Dynamic equation to discuss its influence on the motion of objects at large scales, and attempts to further reveal the deep physical significance of the basic constants a_0 in MOND theory.

2. Newtonian dynamic equation with dissipative potential energy

In the frame of non-relativistic, the Newtonian dynamic equation is

$$\mathbf{F} = m\mathbf{a} \quad (1)$$

Where m is the mass of the object and \mathbf{a} is the kinematic acceleration.

There are usually two ways to modify the Newtonian dynamic equation: the first is to modify the right side of the equation to establish a modified inertia theory, and the second is to modify the left side of the equation to establish a modified gravity theory. In the case of relativity, the two modified theories are equivalent. While, in non-relativistic case, and applying the modified inertia theory, Eq. (1) can be assumed to be the following modification form

$$\mathbf{F} = m(\mathbf{a} - \lambda\mathbf{v}) \quad (2)$$

Where λ is a constant, \mathbf{v} is the velocity of a moving object.

For an isolated gravitational system Σ , consisting of only two objects, and an object with mass m is moving within a 2D-plane in the gravitational field of a quasi-stationary object with mass M ($M \gg m$), we can obtain the Lagrangian equation for system Σ based on Eq. (2)

$$L = E - U = \frac{1}{2}m[(\dot{r})^2 + (r\dot{\theta})^2] + \frac{GMm}{r} \quad (3)$$

Where (r, θ) is the polar coordinate system of the 2D-plane, G is the gravitational constant.

As a velocity-dependent dissipation function is introduced in Eq. (2), then the corresponding dissipation potential is

$$\Psi = -\frac{1}{2}m\lambda[(\dot{r})^2 + (r\dot{\theta})^2] \quad (4)$$

According to the Hamiltonian principle of a dissipative system, the motion equation of the object with mass m is

$$\begin{cases} \frac{d}{dt}(r^2\dot{\theta}) = \lambda r^2\dot{\theta} \\ \ddot{r} - r\dot{\theta}^2 + \frac{GM}{r^2} = \lambda\dot{r} \end{cases} \quad (5)$$

When the object with mass m moves along the radial direction of the polar coordinate system, namely $\theta=0$. Thus, the second formula of Eq. (5) can be rewritten as

$$\ddot{r} = -\frac{GM}{r^2} + \lambda\dot{r} \quad (6)$$

As we known that the cosmological equations can be obtained from Newtonian mechanics [37]. For a sphere with the radius of r , if the physical coordinate $r=a(t)R$ (R is the comoving coordinate of the cosmic expansion, $a(t)$ is the cosmic expansion factor), the density is $\rho=\rho_0 a^3$, and the quality of the material inside the sphere is $M=4\pi/3\rho r^3$, it can be expressed from Eq. (6) that

$$\ddot{a} = -\frac{4\pi G \rho_0}{3a^2} + \lambda \dot{a} \quad (7)$$

Substitute Hubble's law $H = \dot{a}/a$ into Eq. (7), we can obtain

$$\ddot{a} = -\frac{4\pi G \rho_0}{3a^2} + \lambda H a \quad (8)$$

Comparing Eq. (8) to the time-time component of the field equation in the Λ CDM model [10], it can be obtained

$$\frac{\Lambda}{3} \sim \lambda H \quad (9)$$

Eq. (9) shows that the Newtonian dynamic equation with inverse dissipation potential can derive the dynamic equations similar to the expansive cosmological Λ CDM model at large scales.

Now we discuss the second case. For a given initial condition at $t=0$, that an object with mass m circle a quasi-stationary object with mass M in its gravitational field, ie $\dot{r}_0 = 0$, $GM/r_0^2 = \dot{\theta}_0^2 r_0$. In a dissipative system, r and $\dot{\theta}$ will change with time. Let's assume $r = r_0 f_1(t)$, $\dot{\theta} = \dot{\theta}_0 f_2(t)$, then the second formula in Eq. (5) can be rewritten as

$$\ddot{r} - f_1 f_2^2 r_0 \dot{\theta}_0^2 + \frac{1}{f_1^2} \frac{GM}{r_0^2} = \lambda \dot{r} \quad (10)$$

$$\ddot{r} + \frac{GM}{r_0^2} \left(\frac{1}{f_1^2} - f_1 f_2^2 \right) = \lambda \dot{r} \quad (11)$$

Considering the initial condition, when the gravitational field is very weak, namely

$$\frac{GM}{r_0^2} \sim 0 \quad (12)$$

Then substituting Eq. (12) into Eq. (11)

$$\ddot{r} \approx \lambda \dot{r} \quad (13)$$

Solving Eq. (13), then the radius r is

$$r = r_0 e^{\lambda t} \quad (14)$$

From the first formula in Eq. (5), it can be obtained that

$$r^2 \dot{\theta} = r_0^2 \dot{\theta}_0 e^{\lambda t} \quad (15)$$

Therefore

$$r \dot{\theta} = r_0 \dot{\theta}_0 \quad (16)$$

As the rotational tangential velocity $V = \dot{\theta} r$, the initial rotational tangential velocity $V_0 = \dot{\theta}_0 r_0$, then

$$V = V_0 \quad (17)$$

It can be indicated from Eq. (14) and Eq. (17) that the radius increases exponentially with time, but the rotational tangential velocity of the object remains the same, which coincides with the asymptotical flat rotational velocity properties typically exhibited by some spiral galaxies [38,39].

3.MOND Model

As we known, M. Milgrom introduced a basic constant a_0 into the MOND theory, which modified the actual acceleration and Newtonian acceleration in Newtonian mechanics as follows [8,9]

$$a_N = a\mu(a/a_0) \quad (18)$$

When $a/a_0 \gg 1$, $\mu(a/a_0) \sim 1$; when $a/a_0 \ll 1$, $\mu(a/a_0) \sim a/a_0$.

From Eq. (12), it shows that when the gravitational field of the initial condition is very weak, the tangential velocity of the object with mass m remains basically unchanged with time, while, the distance r could still increase. Suppose the intensity of the weak gravitational field at initial corresponds to g_0 , that is

$$g_0 = \frac{GM}{r_0^2} \quad (19)$$

Substituting Eq. (19) into Eq. (17), it can be obtained

$$V^4 = V_0^4 = GMg_0 \quad (20)$$

Rewriting Eq. (20)

$$\lg(L) = 4\lg(V) - \lg(Gg_0 < M/L >) \quad (21)$$

Where M/L is the mass-to-light ratio.

We can see that Eq. (21) is just the Tully-Fisher relationship. Comparing Eq. (20) with the MOND theory, it can be found that $g_0 = a_0$. Therefore, the centripetal/actual acceleration of the object obtained from Eq. (14) is

$$a = \frac{V^2}{r} = \frac{V_0^2}{r_0 e^{2\lambda t}} = \frac{g_0}{e^{2\lambda t}} = \frac{a_0}{e^{2\lambda t}} \quad (22)$$

The Newtonian acceleration is

$$a_N = \frac{GM}{r^2} = \frac{GM}{r_0^2 e^{4\lambda t}} = \frac{g_0}{e^{4\lambda t}} = \frac{a_0}{e^{4\lambda t}} \quad (23)$$

Divide Eq. (23) by Eq. (22)

$$\frac{a_N}{a} = \frac{1}{e^{2\lambda t}} = \frac{a}{a_0} \quad (24)$$

Hence, comparing Eq. (24) with Eq. (18), it shows that the model in this paper can derive the form of the empirical function μ introduced in the MOND theory.

4.Conclusion

An isolated gravitational system is studied in this paper through introducing the inverse dissipation potential into Newton's dynamic equation. It is found that the modified dynamic equation can simultaneously deduce the cosmic expansion dynamic equation similar to the Λ CDM model and the asymptotical flat rotational velocity properties typically exhibited by spiral galaxies, which is generally considered to be dark energy and dark matter, respectively. It suggests that the dark energy effect and the dark matter effect may be two aspects of the same physical reality after the introduction of the inverse dissipation potential.

Unlike the usual dark matter model, this kind of the asymptotic flat rotational velocity properties of the spiral galaxies are the result of long-term accumulations

with time, rather than the direct action effect of mechanics. And in its derived cosmic expansion dynamic equation, an equivalent time-varying cosmological constant is presented, which is consistent with the Quintessence model and the phantom model.

Finally, same with the MOND model, the modified Newton's dynamic equation also derives the Tully-Fisher relationship. Moreover, the basic constant a_0 introduced in the MOND model and the form of the function μ show a clearly corresponding physical significance in the model of this paper. However, the discussion in this paper is still not a relativistic correction of the Newtonian gravity, which requires further study.

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